LETTER TO THE EDITOR. REMARK CONCERNING THE BOOK "LECTURES ON ANALYTICAL MECHANICS" (Fizmatgiz, Moscow, 1960)

PIS'MO V REDAKTSIIU. ZAMECHANIE PO KNIGE "LEKTSII PO ANALITICHESKOI MEKHANIKE" (Fizmatgiz, Moskva, 1960)

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The attention of the readers should be directed to the fact, that the formulation of the theorem on asymptotic stability on p. 290 contains an error. The theorem asserts, that the state of equilibrium of a definite-dissipative system is asymptotically stable, without mentioning that a given position of equilibrium is an isolated one (there are no other states of equilibrium in its vicinity). Without this remark the theorem is erroneous. The proof of the theorem should be changed.

From the condition of the definiteness of the dissipation of the system it follows, that the derivative of the total energy $dE/dt = 0$ for the set M of points in the space of state, for which all $\dot{q}_i = 0$. But then, as is known, using the continuous dependence of the solutions of differential equations on the initial values, one can show that for sufficiently small $|q_i^0|$, $|\dot{q}_i^0|$ all integral curves (of motion) approach asymptotically the largest invariant set T contained in M. In the given case the set T can consist only of states of equilibrium, because they are the only motions contained in M. Therefore, in view of the supplementary condition of the theorem, the set T is reduced to a single point – the given equilibrium state – and the theorem is proved.

As far as the proof of Liapunov's theorem on p. 205 is concerned, it is derivable from the earlier (uncorrected) proof of the theorem on p. 200, but with a change in notation (instead of E – the difference $V - V_0$, instead of $(q, \dot{q})$ – the vector x must be used).

I take this opportunity to express my gratitude to M.A. Galakhov, a student at the Moscow physical-technical Institute, for calling my attention to the error in the theorem.

Translated by G.H.